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We show that in the presence of large scale primordial hypermagnetic fields, it is possible to generate a large amount of CP violation to explain the baryon to entropy ratio during the electroweak phase transition within the standard model. The mechanism responsible for the existence of a CP violating asymmetry is the chiral nature of the fermion coupling to the background field in the symmetric phase which can be used to construct two-fermion interference processes in analogy to the Bohm-Aharanov effect. We estimate that for strong hypermagnetic fields $B_Y = (0.3 - 0.5)\ T^2$ the baryon to entropy ratio can be $\rho_B/s = (3-6) \times 10^{-11}$ for slowly expanding bubble walls.

PACS numbers: 12.15.–y, 98.80.Cq

The understanding of the mechanism that generates the observed excess of baryons over antibaryons in the universe represents one of the most challenging problems for particle physics as applied to cosmology. Any theory that aims to explain such excess has to meet the three well know Sakharov conditions [1], namely: (1) Existence of interactions that violate baryon number; (2) C and CP violation; (3) departure from thermal equilibrium. It is well known that the above conditions are met in the standard model (SM) of electroweak interactions if the electroweak phase transition (EWPT) is of first order. This raised the interesting possibility that the cosmological phase transition responsible for particle mass generation –that took place at temperatures of order 100 GeV– could also explain the generation of baryon number. Consequently, a great deal of effort was devoted to explore such a possibility [2]. Nowadays, the consensus however is that the minimal SM as such, cannot explain the observed baryon number. The reason is that the EWPT turns out to be only too weakly first order which in turn implies that any baryon asymmetry generated at the phase transition was erased by the same mechanism that produced it, *i.e.* sphaleron induced transitions [3]. Moreover, the amount of CP violation coming from the CKM matrix alone cannot account by itself for the observed asymmetry, given that its effect shows up in the coupling of the Higgs with fermions at a high perturbative order [10], giving rise to a baryon to entropy ratio at least ten orders of magnitude smaller than the observed one.

Nevertheless, it has been recently pointed out that, provided enough CP violation exists, the above scenario could significantly change in the presence of large scale primordial magnetic fields [5,6] (see however Ref. [7]), which can be responsible for a stronger first order EWPT. The situation is similar to a type I superconductor where the presence of an external magnetic field modifies the nature of the superconducting phase transition due to the Meissner effect.

Though the origin and nature of these primordial fields is a subject of current research, its existence prior to the EWPT epoch can certainly not be ruled out [8]. The only constraint on their strength comes from considerations on the measured anisotropies of the cosmic microwave background and on nucleosynthesis calculations [9].

Recall that for temperatures above the EWPT, the SU(2)$\times$U$(1)_Y$ symmetry is restored and the propagating, non-screened vector modes that represent a magnetic field correspond to the U$(1)_Y$ hypercharge group. Thus, in the unbroken phase, any primordial magnetic fields belong to the hypercharge group instead of to the U$(1)_{em}$ group and are therefore properly called *hypermagnetic* fields.

In this paper we show that the existence of such primordial hypermagnetic fields also provides a mechanism to produce a large enough amount of CP violation during the EWPT to possibly explain the observed baryon to entropy ratio in the SM. This can happen during the reflection of fermions off the true vacuum bubbles nucleated during the phase transition through an interference process equivalent to the Bohm–Aharanov effect, given that in the unbroken phase, fermions couple chirally to hypermagnetic fields with the hypercharge. The chiral nature of this coupling implies that it is possible to build a CP violating asymmetry dissociated from non-conserving baryon number processes that can then be converted to baryon number in the unbroken phase where sphaleron induced transitions are taking place with a large rate. The existence of such asymmetry provides a bias for baryon over antibaryon production. In the absence of primordial magnetic fields, this mechanism has been proposed and studied in Refs. [10–12] in extensions of the SM.

To describe the EWPT, we start by writing the effective, finite temperature Higgs potential which, including all the one-loop effects and ring diagrams, looks like

$$V_{\rm eff}\,(\phi,T) = \frac{\gamma}{2}(T^2 - T_c^2)\ \phi^2 - \delta\ T\ \phi^3 + \frac{\lambda}{4}\ \phi^4\,, \qquad (1)$$

where $\phi = \sqrt{2}(\Phi^\dagger\Phi)^{1/2}$ is the strength of the SU(2) Higgs doublet Φ whose vacuum expectation value v is given by

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}} \,. \tag{2}$$

The parameters γ, δ and λ have been computed perturbatively to one loop and can be expressed in terms of v, the SU(2) gauge boson masses and the top mass. Their explicit expressions can be found elsewhere (see for example Ref. [6]). δ is the parameter responsible for the first order nature of the phase transition. It is the parameter that gets enhanced in the presence of hypermagnetic fields. T_c is the critical temperature at which spinodal decomposition proceeds.

We can write the effective potential in a more transparent form [13] by introducing the dimensionless temperature ϑ and the dimensionless Higgs field strength φ

$$\begin{aligned} \vartheta &= \frac{\lambda\,\gamma}{\delta^2}\left[1-\left(\frac{T_c}{T}\right)^2\right] \\ \varphi &= \frac{\lambda}{\delta\;T}\phi \,, \end{aligned} \tag{3}$$

in terms of which, the effective potential, Eq. (1), becomes

$$V_{\rm eff}\;(\varphi) = \delta\;T\left(\frac{\delta\;T}{\lambda}\right)^3\left(\frac{\vartheta}{2}\varphi^2 - \varphi^3 + \frac{1}{4}\varphi^4\right) \,. \tag{4}$$

For simplicity, we work in the approximation where the energy densities of both the unbroken and broken phases are degenerate. This happens for a value of $\vartheta = 2$. In this approximation, the phase transition is described by a one-dimensional solution for the Higgs field, called the *kink*, which separates the two phases. This is given by

$$\varphi(x) = 1 + \tanh(x) \,, \tag{5}$$

where the dimensionless position coordinate x is

$$x = \frac{\delta\;T}{\sqrt{2\lambda}}\;r \,. \tag{6}$$

The parameter $\sqrt{2\lambda}/(\delta\;T)$ represents the width of the domain wall [14]. It can also be checked that this parameter becomes smaller in the presence of hypermagnetic fields.

In terms of the kink solution we can see that $x = -\infty$ represents the region outside the bubble, that is the region in the symmetric phase. Conversely, for $x = +\infty$, the system is inside the bubble, that is in the broken phase. The kink wall propagates with a velocity determined by its interactions with the surrounding plasma. This velocity can be anywhere between 0.1–0.9 the speed of light [15].

Given the above background scalar field, the problem of fermion reflection and transmission through the domain wall in the presence of an external magnetic field can be cast in terms of solving the Dirac equation. In the wall's rest frame, this can be written as

$$\left(\not p\;-a\;\not A - \frac{\delta\;T}{\sqrt{2\lambda}}\;\xi\;\varphi(x)\right)\Psi(\mathrm{x}) = 0 \,, \tag{7}$$

where $\xi = 2m/m_H$, m and m_H being the fermion and Higgs masses at zero temperature, respectively and Ψ the fermion wave function. Also, $A^\mu = (0,\mathbf{A})$ is the four-vector potential related to the external magnetic field by $\mathbf{B} = \nabla\times\mathbf{A}$. In the symmetric phase, $\mathbf{A} = \mathbf{A}_Y$ corresponds to the hypermagnetic field vector potential whereas in the broken phase, where only the Maxwell projection is unscreened [5,6], $\mathbf{A} = \mathbf{A}_{em}$ corresponds to the ordinary photon vector potential. a is the fermion coupling to the external field; $a = g'\mathsf{Y}/2$ in the symmetric phase with Y the fermion's hypercharge and g' the $\mathrm{U}(1)_Y$ coupling constant; $a = e$, the electric charge, in the broken phase.

In the absence of a background magnetic field, the solution to Eq. (7) when considering the bubble wall as a planar interface has been found in Ref. [16]. Let $\tilde\Psi_h(\mathrm{x})$ be a solution with a definite helicity ($h = L, R$) to Eq. (7) in the absence of an external magnetic field –since we will be interested in fermion solutions in the symmetric phase where particles are massless, helicity is a good quantum number. It can easily be shown that in the presence of a magnetic field, the solution to Eq. (7) with definite helicity is given by

$$\Psi_h(\mathrm{x}) = \tilde\Psi_h(\mathrm{x})e^{-i\frac{g'}{2}\mathsf{Y}_h\int^{\mathbf{X}}\mathbf{A}_Y\cdot d\mathbf{x}'} \tag{8}$$

where Y_h is the hypercharge of the corresponding helicity mode.

Since the presence of the hypermagnetic field shows up as a phase in the fermion wave function, it follows that its effect will dissapear when considering a process that involves the square of a single fermion wave function. However, the effect will be revealed during an interference process. Since this requires that at least two particles be present within a spatial volume of order λ^3 per unit time, where λ is the de Broglie fermion wave length, we expect that these processes are supressed with respect to those involving a single particle. We will come back later to estimate the probability for such processes. For the time being, let us consider a situation in which the fermion amplitude at a space point a distance l away from the the bubble wall receives contributions from fermions coming from the symmetric phase which are reflected from two points on the bubble wall separated by a distance d. For simplicity, we take $d \ll l$, in this case, we can consider that the fermion propagation is approximately directed along the direction perpendicular to the bubble wall. Since the coupling of the given helicity mode with the hypermagnetic field is chiral, the square of the amplitude representing the interference of left-handed fermions $|M_L|^2$ will differ from the square of the amplitude for right-handed interfering antifermions $|\bar M_R|^2$ and consequently, an axial asymmetry $\mathcal{A}$ is built in front of the bubble wall. Within the above approximations, it is easy to show that the explcit expression for the axial, CP violating asymmetry is given by

$$\begin{aligned}\mathcal{A} &\equiv |M_L|^2 - \ |\bar{M}_R|^2 \\ &= \mathcal{R}\sin(\Upsilon_+/2)\sin(\Upsilon_-/2)\,,\end{aligned} \tag{9}$$

where

$$\Upsilon_\pm = \frac{g'}{2}\left(\mathsf{Y}_R \pm \mathsf{Y}_L\right)\Theta\,. \tag{10}$$

$\mathcal{R}$ is the fermion reflection coefficient [16], common to left and right-handed modes and depends on the parameter ξ and the fermion energy, being equal to 1 for energies below twice the zero temperature mass of the fermion. Θ is the hypermagnetic field flux through the area defined by the two trajectories of the interfering fermions. Taking the case of a constant hypermagnetic field perpendicular to this area, $\Theta = B_Y l d/2$, with $B_Y = |\mathbf{B_Y}|$. Form Eq. (9) we see that the asymmetry disappears for a vanishing hypermagnetic field.

We now follow Ref. [12] and construct the net axial charge flux $\mathcal{F}$ in front of the expanding bubble wall in the symmetric phase. This flux receives contributions both from the reflected fermions within the symmetric phase and from those fermions transmitted from the bubble's interior. From CPT conservation, fermion transmission from the broken to the symmetric phase is related to fermion reflection in the symmetric phase. Therefore, the explicit expression for $\mathcal{F}$ is given by [17]

$$\begin{aligned}\mathcal{F} = \frac{1}{2\pi^2\gamma}&\int_0^\infty dp_l \int_0^\infty dp_t\ p_t \\ &\left[\ f^s(-p_l,p_t) - f^b(p_l,p_t)\ \right]\ \mathcal{A}\,,\end{aligned} \tag{11}$$

where the bubble's wall velocity in the fluid frame is u and $\gamma = 1/\sqrt{1-u^2}$ is the Lorentz factor. p_l, p_t are the longitudinal and transverse components of the fermion's momentum and f^s, f^b are the fermion equilibrium fluxes (neglecting Fermi blocking factors) in the symmetric and broken phases, respectively

$$\begin{aligned} f^s &= \tfrac{p_l/E^s}{\exp[\gamma(E^s - up_l)/T]+1}, \ \ E^s = \sqrt{p_l^2+p_t^2} \\ f^b &= \tfrac{p_l/E^b}{\exp[\gamma(E^b + up_l)/T]+1}, \ \ E^b = \sqrt{p_l^2+p_t^2+m^2}\,. \end{aligned}$$

Since there is no net fermion number flux through the wall, the axial charge $\mathcal{F}$ and hypercharge $\mathcal{F}_Y$ fluxes are trivially related by $\mathcal{F}_Y = \mathcal{F}/4$.

The rate of baryon number density production is given in terms of the rate of baryon number violation per unit volume Γ_B and the partial derivative of the free energy with respect to baryon number $\partial F/\partial B$ by [10]

$$\dot{\tilde{\rho}}_B = -\frac{\Gamma_B}{T}\frac{\partial F}{\partial B}\,. \tag{12}$$

The quantity $\partial F/\partial B$ corresponds to the baryon number chemical potential μ_B and represents the force pushing the universe towards its equilibrium baryon number value. Only those processes which happen fast enough with respect to the time that the reflected fermions in the symmetric phase (or those that passed from the broken to the symmetric phase) spend before being retaken by the expanding wall, will contribute to drive baryon number towards equilibrium. This time is called the *transport time* τ and estimates show [12] that it is of order $\tau \sim 100/T$. Since the only other interactions, besides SU(3)$\times$U(1) gauge boson exchange and family-diagonal SU(2) gauge boson exchange, with a large enough rate are the top quark Yukawa interactions (given the large top Yukawa coupling), then the axial asymmetry must reside basically in the form of top quarks. Enforcing this condition together with that of having initially zero net baryon and lepton numbers one finds [12]

$$\mu_B = \frac{\partial F}{\partial B} = -\frac{4\rho_Y}{(1+2n)T^2}\,, \tag{13}$$

where ρ_Y is the hypercharge density, n is the number of scalar doublets in the theory and the minus sign comes from the assumption that the net axial asymmetry is in the form of a right-handed top number. For the SM, $n=1$.

It is now straightforward to integrate Eq. (12) to get the net baryon number density in terms of the axial flux, with the result [12]

$$\tilde{\rho}_B = \frac{\Gamma_B}{3T^3}\frac{\tau}{u}\mathcal{F}\,, \tag{14}$$

Equation (14) has to be corrected by recalling that the axial asymmetry, Eq. (9), is built from two-fermion interfering processes. We estimate the probability η of finding a second fermion in the trajectory of the first one by assuming that there is no initial correlation between these two particles. The probability to find the second one is thus given by the ratio of the number of particles per unit area and time that bounce off the bubble wall making an angle θ within a solid angle subtended by a linear dimension on the order of the particles wave length ($\lambda \sim T^{-1}$) to the incoming flux. θ is such that

$$\cos\theta = \frac{l}{\sqrt{l^2+(d/2)^2}}\,. \tag{15}$$

Taking p, $dp \sim T$ and neglecting Fermi blocking factors, η is given by

$$\eta = \frac{2}{3\pi\zeta(3)}\frac{\lambda^2\ l\ d/2}{[\ l^2+(d/2)^2\ ]^2}\frac{1}{1+e}\,, \tag{16}$$

where ζ is the Riemann zeta function and the last factor comes from a simple Fermi-Dirac distribution evaluated at $E=T$.

Notice that in the wall's rest frame, the interference point cannot be further apart than the particle's mean free path $\lambda_{\rm mfp}(\sim (1-10)/T$ [14]), since otherwise fermions will rethermalize and no net baryon number could be produced. Accounting for this probability factor, the net baryon to entropy ratio produced during the EWPT can be written as

$$\begin{aligned}\rho_B/s &= \eta\tilde{\rho}_B/s \\ &= \eta\frac{270\kappa\alpha_{\mathrm{W}}^4}{12\pi^2 g_\star}\frac{\tau\mathcal{F}}{uT^2}\,,\end{aligned} \tag{17}$$

where we have used that for the EWPT epoch, the entropy density is given by $s = 2\pi^2 g_\star T^3/45$ with the effective number of degrees of freedom $g_\star \simeq 107$ and that the rate of baryon number violation per unit volume in the symmetric phase is given by

$$\Gamma_B = 3\kappa\alpha_{\mathrm{W}}^4 T^4\,, \tag{18}$$

with $\alpha_{\mathrm{W}} = g^2/4\pi$, g the SU(2) coupling constant and $\kappa \sim 1$.

Equation (17) is valid when the rate of baryon number violation is much smaller than the rate at which the fermions are recaught by the expanding wall, which in turn sets a lower limit for the wall's velocity given by

$$3\kappa\alpha_w^4 T\tau \ll u\,. \tag{19}$$

For definitiveness, we set $\kappa = 1$, $m_h = T = 100$ GeV and $d = l/2$. (For the SU(2) and $\mathrm{U}(1)_Y$ couplings, we use $g = 0.637$ and $g' = 0.344$, respectively [18]). We find that for a wall velocity $u = 0.1c$

$$\rho_B/s = (3-6)\times 10^{-11} \tag{20}$$

whereas for $u = 0.6c$

$$\rho_B/s = (0.7-1.3)\times 10^{-11}\,, \tag{21}$$

for $l \sim 9/T$ and $B_Y = (0.3-0.5)\,T^2$. These are large but still not ruled out values for the background field strength [9], compatible with a Higgs mass of order 100 GeV and a phase transition analog to a type I superconductor [6]. Recall that the experimental value based on nucleosynthesis calculations [19] is

$$\rho_B/s = (0.1-1.4)\times 10^{-10}\,, \tag{22}$$

which means that the estimates based on the present analysis are within the experimental limits, at least for slowly expanding bubbles.

In conclusion, we have shown that in the presence of strong, large scale primordial hypermagnetic fields, it is possible to generate a large amount of CP violation that combined with a stronger first order EWPT –also produced by the hypermagnetic fields– could account for the observed baryon number to entropy ratio within the SM. The fact that fermions couple chirally to background hypermagnetic fields in the symmetric phase makes it possible to build a CP violating asymmetry by considering two fermion interfering processes in an equivalent way to the Bohm-Aharanov effect. This asymmetry is converted into baryon number by sphaleron induced processes in the symmetric phase and preserved when these fermions are recaught by the expanding bubble wall.

Given that some of the parameters describing the dynamics of the EWPT are not very precisely determined, it is clear that work on that direction is necessary and the possibility that baryogenesis could be realized within the SM could certainly stimulate this kind of work.

Support for this work has been received in part by CONACyT-Mexico under grant numbers 32279-E and 29273-E.

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